

EVALUATION OF CUT-OFF FREQUENCY AND CORRECTION OF FILTER-INDUCED PHASE LAG AND ATTENUATION IN EDDY COVARIANCE ANALYSIS OF TURBULENCE DATA

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(Received in final form 28 October 2002)

Abstract. A recursive filter adopted for online eddy covariance analysis of turbulence data in the atmospheric surface layer is revisited, and its properties and performance are evaluated by means of concepts and methods developed in digital signal analysis. A rigorous estimate of effective cut-off frequency is derived along with an estimate of filter induced phase lag. Accordingly, filter design criteria are revised and various effects of parameter choice are assessed. Furthermore suitable corrections for compensating filter induced distortion, i.e., phase lag and attenuation, are proposed. The modified filter is tested on: (a) An artificial time series (random noise superimposed to 'slow' sinusoidal signal); (b) turbulence data from field measurements. In case (a) the retrieval of input signal parameters is appreciably improved. In case (b) a better agreement with a Gaussian moving average is obtained, at lower computational cost.

Keywords: Atmospheric surface layer, Phase lag, Recursive filtering, Turbulence.

1. Introduction

The eddy correlation technique has become widely used for the analysis of turbulence data in the atmospheric surface layer (ASL). One of the most frequently applied procedures has been introduced by Lloyd et al. (1984) and McMillen (1988), who provided a rather simple method to extend the applicability of the eddy correlation technique to turbulent flows over complex terrain.

Besides the mean wind alignment procedure, which is commonly and widely adopted in ASL studies (McMillen, 1988; Kaimal and Finnigan, 1994), the key point of the above method is the use of a first-order recursive filter for the online detrending of the raw dataset. The advantage of recursive procedures in comparison to other averaging algorithms, such as weighted moving average or block average, is the lower computational cost and reduced memory requirements.

Although the continuous increase of the storage capacities allows the postprocessing of raw data and then filtering without any phase lag, in order to estimate the eddy covariances some applications, such as the relaxed eddy accumulation

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(Beverland et al., 1996, 1997; Moncrieff et al., 1998, Gallagher et al., 2000), still require real-time high-pass filtering.

The mathematical formulation of the low-pass recursive filter proposed by McMillen (1988) is given by the simple relationship

$$y_n = \alpha y_{n-1} + (1 - \alpha) x_n, \quad (1)$$

$$x_n^{\text{filt}} = x_n - y_n, \quad (2)$$

where y_n is the value of the filter at time t_n , x_n is the value of the input time series at time t_n , y_{n-1} is the filter at time $t_{n-1} = t_n - \delta t$ (δt is the sampling interval), x_n^{filt} is the high frequency component and the coefficient α is given by

$$\alpha = \exp\left(-\frac{\delta t}{\tau}\right), \quad (3)$$

where τ is the time constant of the filter.

Since usually $\delta t/\tau \ll 1$, Equation (3) can be approximated by means of a truncated Taylor series expansion:

$$\alpha \cong 1 - \frac{\delta t}{\tau}. \quad (4)$$

The design parameter τ has to be related to the required cut-off frequency of the filter, i.e., it specifies the low frequencies associated with mean flow. McMillen (1988) suggested, on the basis of a sensitivity test, a value of 200 s as adequate for the dataset used. The same value was also adopted by Rannik (1998), while later Rannik and Vesala (1999) showed that higher values (around 1000 s) avoided systematic underestimation of fluxes but overestimated the variances. This feature was mainly related to a lagging of the running mean from the trend, thus increasing random errors during highly non-stationary events, and therefore the uncertainty of the short-term flux estimates.

In this paper we study in detail the problem following a different approach: rather than concentrating on the cross-covariance functions and one-sided co-spectral density functions for the evaluation of the systematic underestimation of the fluxes, we consider first-order recursive filter in the framework of digital signal processing (Mittra, 1998), obtaining a simple correction formula that allows a rigorous estimate of the low-frequency component of turbulent signal.

Therefore the paper is organised as follows. In Section 2 an overview on mathematical concepts for analytical formulation of the discrete-time series filtering procedures is provided. This allows the evaluation of transfer functions and rigorous setting of *cut-off frequency* and *phase delay*. Suitable tests of the procedure on selected time series are shown in Section 3, and in Section 4 some conclusions along with possible further developments are discussed.

2. Mathematical Formulation

Consider a discrete time series where x_n is the datum at time $t = n \delta t$, where δt is the sampling interval. A common way of representing the input-output relationship of a discrete-time system, such as the recursive filter reported in Equation (1), refers to the frequency response function. This provides a frequency-domain description of the system (Mitra, 1998).

To this purpose, the discrete-time Fourier transform of a general sequence x_n is introduced

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_n e^{-j\omega n}. \quad (5)$$

Equation (5) is generally a complex function of the nondimensional angular frequency $\omega = 2\pi \delta t / \mathcal{T}$ (\mathcal{T} is a time period) and can also be written as

$$X(e^{j\omega}) = |X(e^{j\omega})| e^{j\theta(\omega)}, \quad (6)$$

where $|X(e^{j\omega})|$ is the magnitude function and

$$\theta(\omega) = \text{arg} \{X(e^{j\omega})\} \quad (7)$$

is the phase function.

The Fourier transform can be defined whenever the sufficient condition of absolute convergence of the sequence is satisfied, namely

$$\sum_{n=-\infty}^{\infty} |x_n| < \infty. \quad (8)$$

In many cases condition (8) is not satisfied, and such a frequency-domain characterization of the filter is not applicable. For this reason it is customary to generalise the Fourier transform leading to the z-transform, which is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x_n z^{-n}, \quad (9)$$

where z is a complex variable. Notice that setting $z = r e^{j\omega}$ in Equation (9) the Fourier transform of the new sequence $\{x_n r^{-n}\}$ is obtained. Also for $r = 1$, or equally $|z| = 1$, the z-transform reduces to the Fourier transform as defined by Equation (5). If the region of convergence of $X(z)$, which is bounded by the locations of its poles, includes the unit circle, then $X(z)$ is related to the frequency response $X(e^{j\omega})$ by

$$X(e^{j\omega}) = X(z)|_{z=e^{j\omega}}. \quad (10)$$

The recursive filter of Equation (1) belongs to the class of finite-dimensional ‘Linear Time Invariant – Infinite Impulse Response’ (LTI-IIR) filters (Mitra, 1998). Their most general form can be expressed by means of a difference equation as

$$\sum_{n=0}^N d_n y_{k-n} = \sum_{n=0}^M p_n x_{k-n}. \quad (11)$$

Finally, we recall that the filtering operator can be considered a convolution process defined as $y_i = h_i \otimes x_i$, where x_i , y_i and h_i are, respectively, the input, output and impulse response sequences. Then taking the z-transform we simply obtain $Y(z) = H(z)X(z)$, which defines the transfer function as $H(z) = Y(z)/X(z)$.

By taking the z-transform of Equation (11) we obtain the following expression for the transfer function

$$\left(\sum_{n=0}^N d_n z^{-n} \right) Y(z) = \left(\sum_{n=0}^M p_n z^{-n} \right) X(z). \quad (12)$$

It can be shown that the general expression for the transfer function of an IIR is

$$H(z) = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + \dots + p_M z^{-M}}{d_0 + d_1 z^{-1} + d_2 z^{-2} + \dots + d_N z^{-N}}. \quad (13)$$

Thus for the first-order filter of Equation (1) the transfer function is simply given by

$$H(z) = \frac{1 - \alpha}{1 - \alpha z^{-1}}. \quad (14)$$

The condition $|\alpha| < 1$ has to be met to satisfy the stability condition.

Recalling Equation (10), the squared magnitude of the response function of the filter is given by

$$|H(e^{j\omega})|^2 = \frac{(\alpha - 1)^2}{1 - 2\alpha \cos \omega + \alpha^2}, \quad (15)$$

where, as previously stated, $\omega = 2\pi \delta t / \mathcal{T}$ is a normalised angular frequency with δt sampling interval and \mathcal{T} the time period.

2.1. CUT-OFF FREQUENCY

The next step is stating a relationship between α and the desired cut-off frequency, which is usually done starting from the usual definition of a gain function (Mitra, 1998)

$$G(\omega) = 20 \log_{10} |H(e^{j\omega})|. \quad (16)$$

It is then customary to define the cut-off angular frequency ω_c as that which determines an attenuation of 3 dB, i.e., by the condition

$$G(\omega_c) = -3 \text{ dB}. \quad (17)$$

Substituting from (15) into (17) it follows, with reasonable approximation (within 0.2%, see Appendix A)

$$|H(e^{j\omega})|^2 \cong 1/2 \quad (18)$$

and,

$$\cos \omega_c = \frac{-\alpha^2 + 4\alpha - 1}{2\alpha}, \quad (19)$$

where $\omega_c = 2\pi \delta t / \tau_c$. Solving Equation (19) for the parameter α leads to

$$\alpha = 2 - \cos \omega_c - \sqrt{\cos^2 \omega_c - 4 \cos \omega_c + 3} \quad (20)$$

(the other possible root leads to an unstable filter displaying $|\alpha| > 1$). Equation (20) provides a unique and rigorous definition of the design parameter α in terms of the cut-off time period, τ_c , defining the time scale of low frequency components of the signal. In fact Equation (20) was already reported by Otnes and Enochson (1972, p. 70, Equation (3.13)).

Notice however that the design parameters δt and τ_c enter through the variable ω_c in an expression different from that proposed by McMillen (1988) [Equations (3) and (4)].

However, assuming $\delta t \ll \tau_c$ (as commonly met), we can approximate Equation (20) obtaining (see Appendix A for details)

$$\alpha \cong 1 - \omega_c = 1 - 2\pi \frac{\delta t}{\tau_c}. \quad (21)$$

This differs from Equation (4) by a factor 2π ! Although the difference between the two expressions is relatively small and becomes greater only for higher values of $\delta t / \tau$ and $\delta t / \tau_c$ respectively, nevertheless it may strongly affect the behaviour of the filter.

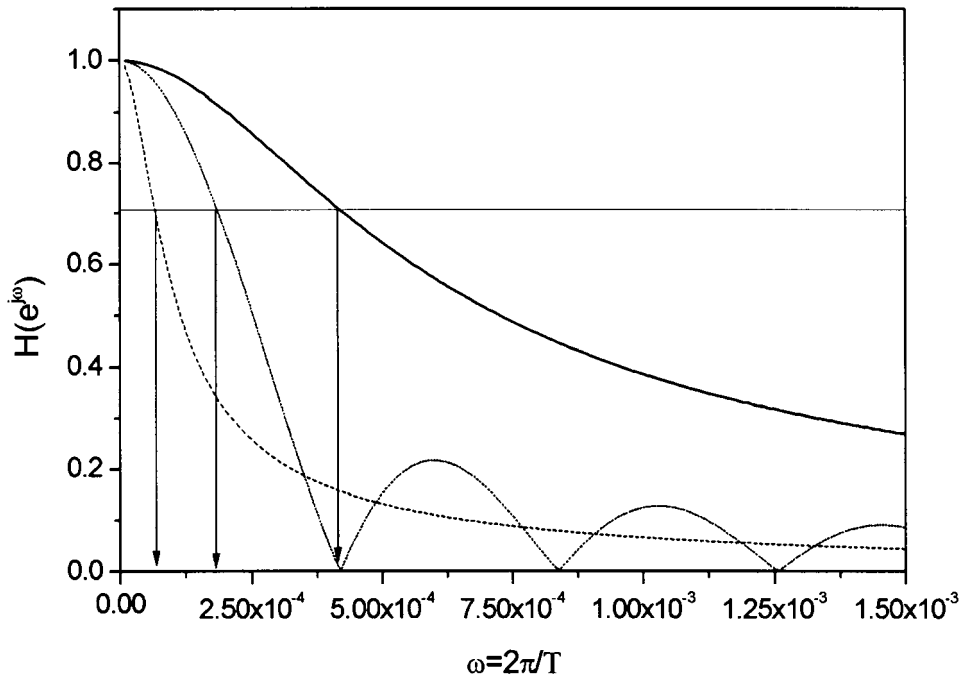


Figure 1. Magnitude response function (Equation (15)) for three different types of filter with a 'design' cut-off period $\tau_c = 300$ s and a sampling frequency 50 Hz. Thick line: Recursive filter with α from Equation (20). Dashed line: Recursive filter with α from Equation (3). Thin line: Box-car moving average. The horizontal line corresponds to $|H(e^{j\omega})| = 1/\sqrt{2}$ which indicates an attenuation of 3 dB and its intersection with the three curves allows the evaluation of the real cut-off period of the filter.

To show this effect we evaluate α from Equations (3) and (20) respectively, assuming typical values of $\delta t = 0.02$ s and $\tau_c = 300$ s. Then the magnitude of the response function in both cases is plotted for comparison in Figure 1.

In the same figure the transfer function of a box-car moving average (which can also be implemented in a recursive way),

$$|H(e^{j\omega})|^2 = \left| \frac{1}{M} \frac{\sin(M\omega/2)}{\sin(\omega/2)} \right|^2, \quad (22)$$

(Mitra, 1998) is also plotted for comparison ($M = \tau_c/\delta t$ is the dimension of the box). The horizontal line corresponding to $|H(e^{j\omega})| = 1/\sqrt{2}$ indicates an attenuation of 3 dB: The intersections of this line with the three curves allow a direct evaluation of the real cut-off period for each filter. It is clear that an incorrect value of the parameter α in Equations (3) and (4) determines a real cut-off frequency much lower than designed (i.e., $\tau_c \cong 1885$ s instead of 300 s).

This is the first source of systematic error in the evaluation of the statistical properties of the signal. A quantitative estimate of the error magnitude is provided by the test reported in Section 3.

2.2. THE PHASE DELAY

A second source of error is the so called *phase delay*. As pointed out by Moncrieff et al. (1998) and Horst (2000), the application of a frequency-dependent filter to a time series leads to a phase shift. However, no estimate has been provided in the literature, to the authors' knowledge, to account for such a dependence.

The phase delay is defined as (Smith, 2002)

$$P(\omega) = -\frac{\theta(\omega)}{\omega} \quad (23)$$

with

$$\theta(\omega) = \arg \{H(e^{j\omega})\} \quad (24)$$

being the phase response of the discrete-time system. Equation (23) represents the *time delay* (in seconds) affecting each harmonic component of frequency ω in the input signal.

Consider an input signal in the form of an amplitude modulated sinusoid (Papoulis, 1977)

$$x_n = a(n \delta t) \cos(\omega_k n \delta t), \quad (25)$$

where $\cos(\omega_k n \delta t)$ is the *carrier wave* and $a(n \delta t) \geq 0$ is the *amplitude envelope*. When this is passed through a filter with a response function $H(e^{j\omega}) = A(\omega) e^{j\theta(\omega)}$, assuming the phase of the filter is linear within the passband, the output is

$$y_n = a[n \delta t - T(\omega_k)] \cos\{\omega_k [n \delta t - P(\omega_k)]\}, \quad (26)$$

where $T(\omega_k)$ is the so-called *group delay* (Papoulis, 1977; Mitra, 1998).

Therefore the phase delay evaluated for the frequency ω_k is a measure of the time delay affecting the carrier, as clearly pointed out by Smith (2002).

With the above definitions, the phase response for the first-order recursive filter of Equation (1) becomes

$$\theta(\omega) = \tan^{-1} \left(-\frac{\alpha \sin \omega}{1 - \alpha \cos \omega} \right), \quad (27)$$

from which it is straightforward to derive from Equation (23) the phase delay

$$P(\omega) = \left\{ \tan^{-1} \left(\frac{\alpha \sin \omega}{1 - \alpha \cos \omega} \right) \right\} \omega^{-1}. \quad (28)$$

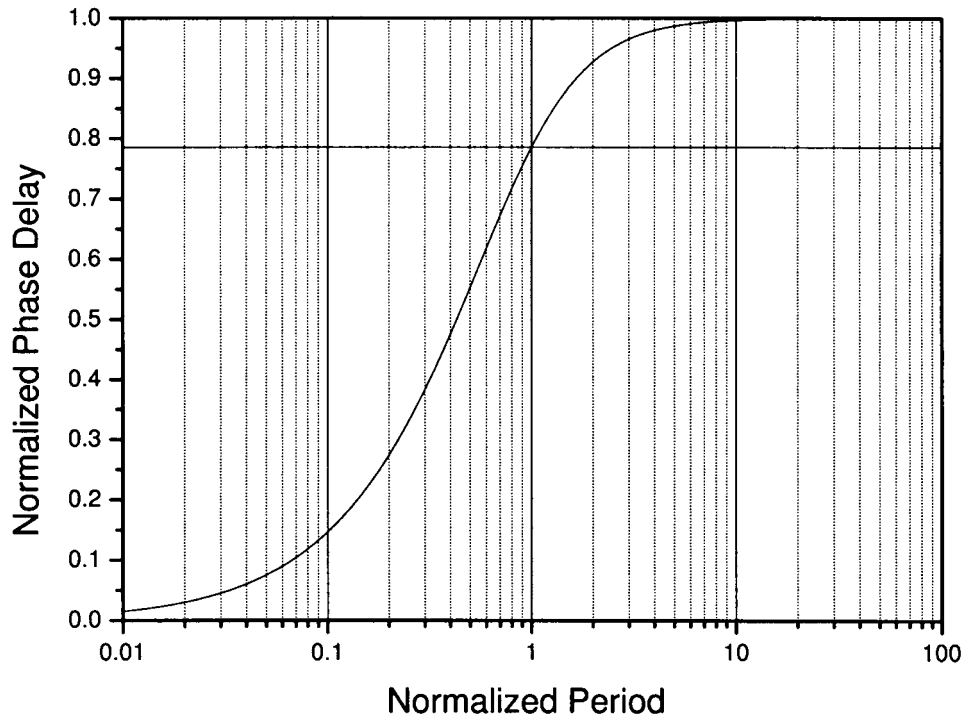


Figure 2. Phase delay (Equation (28)) normalised with the asymptotic value (Equation (29)) versus normalised period \mathcal{T}/τ_c . The horizontal line intersects the curve at $P(\omega_c)$.

This frequency-dependent quantity is mainly responsible for the lagging of the low frequency trend reported among others by Rannik and Vesala (1999).

To compensate for this effect, a suitable correction should be applied to each harmonic component of the input signal. However a good result can be more easily obtained by simply shifting in time the whole low frequency time series by the quantity $P(\omega_c)$. In fact for large values of \mathcal{T} (i.e., $\omega \rightarrow 0$) the phase delay provided by Equation (28) approaches the asymptotic value

$$\lim_{\omega \rightarrow 0} P(\omega) = \frac{\alpha}{1 - \alpha}. \quad (29)$$

We normalise the phase delay with respect to the above asymptotic value, and plot against nondimensional period \mathcal{T}/τ_c in Figure 2. It is clear that all the signal components displaying periods longer than the cut-off period (i.e., $\mathcal{T} > \tau_c$) are almost equally delayed. Moreover components displaying periods lower than τ_c (i.e., normalised periods < 1) are weakly affected by the phase delay. This seems to provide a rational basis for use of $P(\omega_c)$, which is related to the design parameters of the filter only, as the best estimate for correction of phase delay.

It may be argued that values of the delay are relatively small for cut-off periods τ_c usually adopted (around 37s for $\tau_c = 300$ s). However, they can make a

significant difference in terms of the number of data points, as the delay could be very large at high sampling rates. This, in turn, can strongly affect the estimation of standard deviations, as will be shown in the following section.

3. Testing the Correction

The suggested corrections are tested on: (a) An artificial time series (random noise superimposed on a ‘slow’ sinusoidal signal); (b) turbulence data from field measurements. Accordingly we use Equation (20) instead of Equation (3) for evaluation of α and apply a suitable backward-in-time shift, evaluated as $P(\omega_c)$, to the low frequency output time series before subtracting it to the original input series to obtain the turbulent signal.

(a) Artificial Time Series

The artificial time series is made of a sinusoidal signal of known base frequency and a superimposed noise with zero mean and known standard deviation σ_{real} . All the parameters of the filter are imposed so as to capture the frequency of the sinusoidal base signal (i.e., the cut-off frequency is the same as the input base frequency).

In order to test the ability of Equation (28) to capture and correct the time delay, we also use the purely sinusoidal time series without any noise. The effective time delay is evaluated as the distance between the peaks of the original time series and the filtered time series. Linear regression of the predicted time delay against the real one over a range of base frequencies ($10^{-3} - 10^{-1}$ Hz) displays a correlation coefficient $R^2 = 0.9989$.

An example of correction effects is shown in Figure 3. Here the thin-dotted curve A is the low-frequency component evaluated from the original formulation by McMillen (1988) as reported in Equations (1) and (3). Both the effects of a strong attenuation of the signal (due to the *wrong* choice of the parameter α) and a time shift can be clearly appreciated. The thick-dashed curve B shows the improvements introduced by the use of Equation (20) for the evaluation of the parameter α , even if a time shift can still be detected. The thick continuous curve C is the result of further phase delay correction reported above.

Running the filters on time series with different base frequencies we estimated the error due to the different effects. Since σ_{real} represents the *true* standard deviation of our time series, the error has been evaluated as the ratio between the standard deviations obtained from the subtraction of the low frequency from the signal and σ_{real} . The results of this procedure are reported in Figure 4. The upper thin-dotted curve A corresponds to the filter expression of Equations (1) and (3) as proposed by McMillen (1988). The thick-dashed curve B represents the error after the correction applied to the α parameter using Equation (20), and the thick

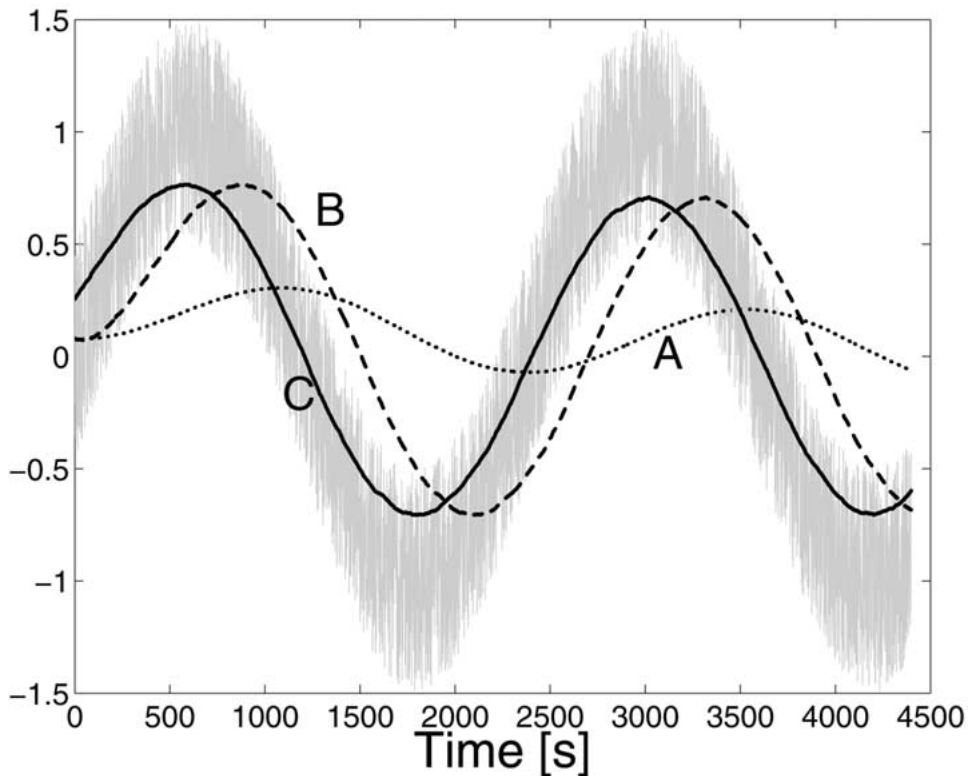


Figure 3. Output of the recursive filters applied to a simple sinusoidal signal with superimposed noise (grey curve). The thin-dotted curve A is the low-frequency component evaluated from the original formulation by McMillen (1988) (Equations (1) and (3)). The thick-dashed curve B uses Equation (20) for the evaluation of the parameter α . The thick continuous curve C is the result of further phase delay correction as reported in Equation (28).

continuous line C is the final result of the correction of the phase delay induced by the recursive form of the filter.

The suggested corrections to the filtering procedure strongly reduce the overestimate of the standard deviations. The remaining error can be ascribed to the smoothing effect of the filter, whose transfer function is not as sharp as a step-shaped response function (cf. thick curve of Figure 1) typical of an ideal filter.

(b) *Turbulence Data*

Similar improvements are obtained by applying the corrected filter to turbulence data from field measurements (de Franceschi et al., 2000). In Figure 5 we plot a 30-min record of one wind component sampled at 50 Hz with an ultrasonic anemometer (Gill Instruments Ltd. Mod. HS Research) along with the low frequency time series evaluated with the original recursive filter (McMillen, 1988: dotted curve A), the corrected recursive filter (thick solid curve C) and a Gaussian

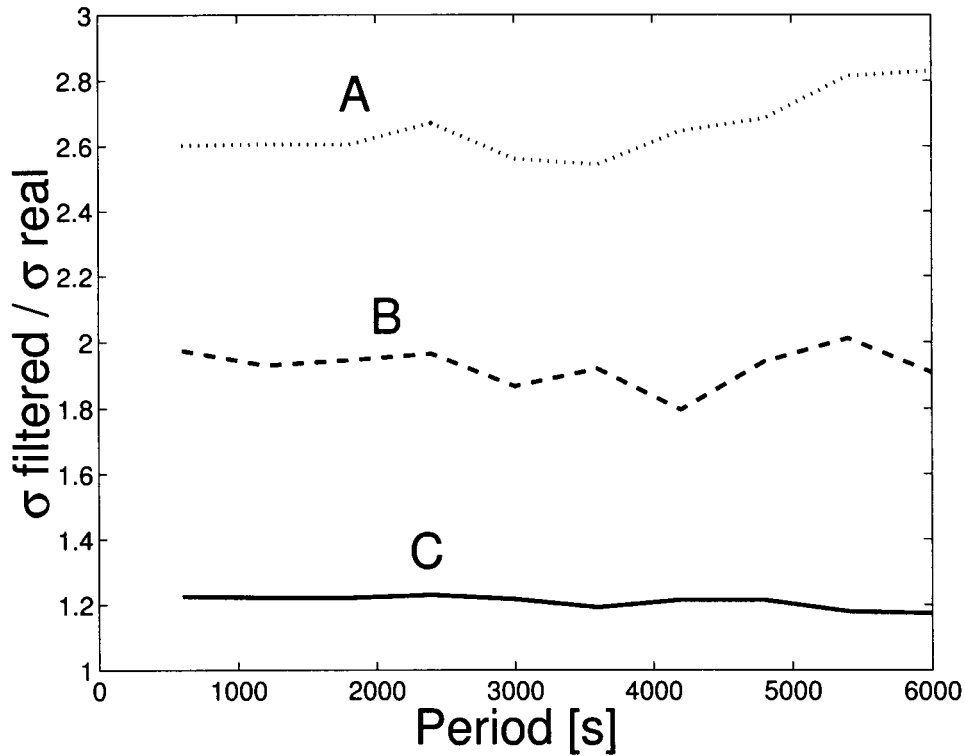


Figure 4. Ratio between calculated and real standard deviations σ of the artificial sinusoidal time series for different values of the input base signal frequency (also set as the cut-off frequency). The upper thin-dotted curve A refers to the original (McMillen, 1988) filter without correction (Equations (1) and (3)); the thick-dashed curve B represents the effect of correcting the α parameter with Equation (20); the lower thick curve C is obtained after further correction of phase delay.

moving average (dash-dotted curve G) respectively, all being designed for a cut-off period $\tau_c = 300$ s. In order to appreciate the improvements to turbulence statistics, we apply all the above filters to the series obtained during 14 days of continuous field measurements. The Gaussian moving average is adopted as a paradigm of spectrally well-behaved offline filter in the time domain, and data from the original McMillen (1988) filter and from the modified one are plotted against it. Results are reported in Figure 6. Only friction velocity and sensible heat flux are shown, the other quantities (such as momentum fluxes, standard deviations) displaying similar behaviour. Regression lines (solid) show how closely the corrected recursive filtered output behaves compared to the Gaussian moving average. On the contrary the original (McMillen, 1988) filter displays discrepancies up to 25%.

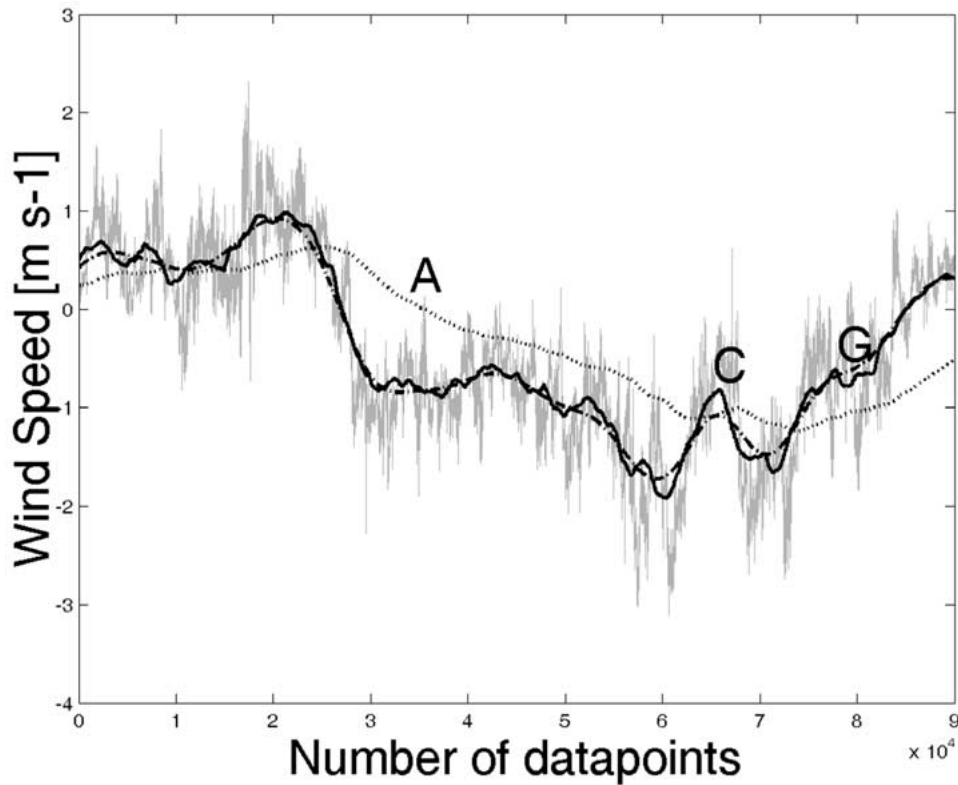


Figure 5. Comparison between the original filter expressed by Equations (1) and (3) for curve A, and the corrected using Equations (20) and (28) for curve C over a real time series (30 min with 50 Hz of sampling frequency). The dash-dotted curve G is the low frequency component evaluated with a Gaussian moving average.

4. Conclusions

The recursive filter proposed by Lloyd et al. (1984) and McMillen (1988) as a suitable procedure for online detrending in eddy covariance measurement systems has been revisited. A modified definition of the coefficient α entering the filter design has been provided on the basis of digital signal analysis. Filter induced phase delay and attenuation have been estimated and suitable correction has been proposed and successfully applied to test cases. The modified procedure reduces the overestimate of standard deviations and provides better agreement with the output of Gaussian moving average accordingly designed.

The scheme provides a rational and unique link of design parameters to the desired cut-off frequency. The latter however still has to be chosen on the basis of physical properties of the process under investigation (especially non stationarities) as displayed by the input time series.

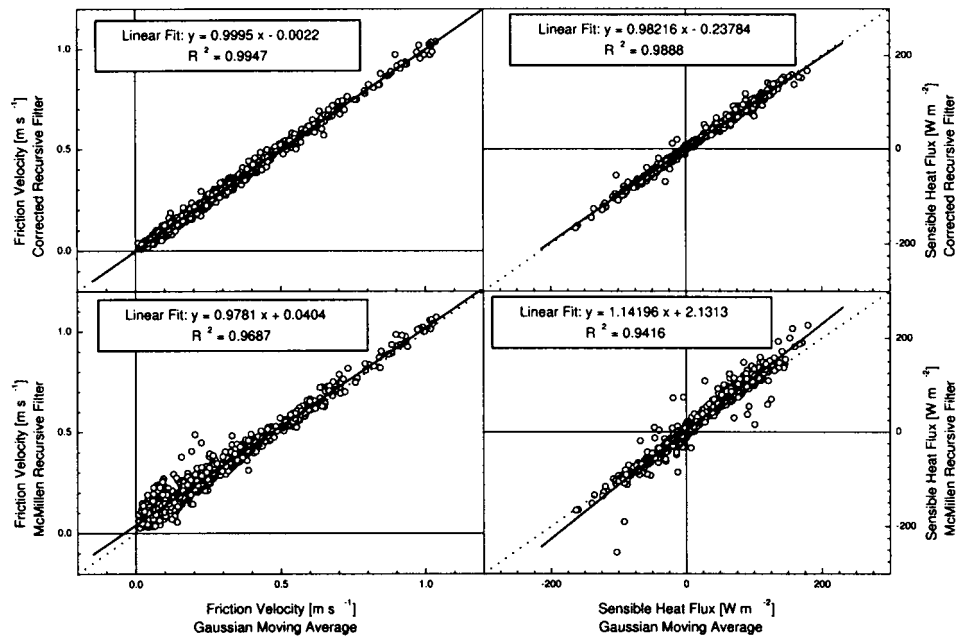


Figure 6. Comparison between the friction velocity (left panels) and the sensible heat flux (right panels) obtained upon application of the corrected recursive filter (upper panels) and McMillen's (1988) recursive filter (lower panels) versus the same quantities obtained upon Gaussian moving average. Thick lines are linear regressions of the data. Dotted lines are 1 : 1 curves.

A crucial test to evaluate the performance of the modified filter will be the comparison with similarity functions proposed in the literature. Better fit of quantities such as turbulent fluxes and variances, obtained by means of this filter with reference functions, will further confirm the filter skill.

The effects of the modified filter on the evaluation of average quantities and turbulence statistics will be the subject of a forthcoming paper.

Acknowledgements

This work has been partly supported by: The City Council of Bolzano (contract 'Evaluation of the pollutant dispersion in the Bolzano area'), the Italian National Institute for Research on the Mountains (INRM, research project 'Study of Atmospheric Dynamics in Alpine Valleys'), the Environmental Protection Agency of the Province of Trento (project 'Study of the atmospheric processes in the Adige Valley') and the Province of Trento (project 'Ora del Garda: atmospheric measurements for the identification of a valley wind system', PAT-UNITN 2001).

Appendix A

The approximation of Equation (18) is easily proved by observing that -3 dB attenuation implies, from Equation (17),

$$|H(e^{j\omega})|^2 = 10^{-3/10} = 0.501187234 \cong 0.5. \quad (\text{A1})$$

The simplified expression of Equation (21) can be obtained from cosine Taylor's series expansion, $\cos \omega_c = 1 - \frac{1}{2}\omega^2 + O(\omega^4)$, valid for $\delta t \ll \tau_c$. Hence

$$\cos^2 \omega_c = 1 - \omega_c^2 + O(\omega^4). \quad (\text{A2})$$

Substituting in Equation (20) leads to

$$\begin{aligned} \alpha &\cong 2 - \left(1 - \frac{1}{2}\omega_c^2\right) - \sqrt{1 - \omega_c^2 - 4\left(1 - \frac{1}{2}\omega_c^2\right) + 3} \\ &= 1 - \omega_c = 1 - 2\pi \frac{\delta t}{\tau_c}. \end{aligned} \quad (\text{A3})$$

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